

1 Induction

1.1 Examples

1. Prove that $2^n > n^2$ for all $n > 4$.

1.2 Problems

2. Prove that $3^n < n!$ for all $n > 6$.
3. Prove that $a_n = 3^n + 1$ for $n \geq 1$ is the solution to the recurrence relation $a_{n+1} = 4a_n - 3a_{n-1}$ with $a_1 = 4, a_2 = 10$.

2 Probability

2.1 Concepts

4. A **probability space** is a pair (Ω, P) where Ω is called the space of outcomes and P is the probability function. An **outcome** is a single thing that could happen in an “experiment”. For example, everything that happened at the super bowl this year. An **event** is usually the thing we care about and can often be described in a single sentence. For instance, it could be “Patriots win” or “Rams only score 3 points”. The probability function P is a function that takes events (or sentences) to the probability of that event happening.

2.2 Examples

5. Assume that a telephone number is a 7 digit number that does not begin with 0 or 1. If I pick a random telephone number, what is the probability that it begins with a 9 or ends with a 0? What is the probability space?
6. In the 3 classes I’m taking, each has 3 HW assignments that have to be done in order (for a total of 9 HW assignments). If I randomly pick an order to do these 9 HWs, what is the probability that I actually do each of the 3 HWs in order?

2.3 Problems

7. True False The probability function P takes outcomes and outputs a probability for that outcome.
8. True False When calculating the probability of an event $A \subset \Omega$, we can always take $P(A) = |A|/|\Omega|$.
9. I roll 4 6-sided die. What is the probability that the sum of the numbers rolled is 7?
10. What is the probability that a 5 card poker hand contains at least 1 king?
11. I am giving out grades to 60 students randomly (A, B, C, D, F). What is the probability that at least half the class got A 's?
12. Out of the 14 pants that I own, there are 5 of them that are white. Every day for two weeks, I randomly put on a new pair of pants. What is the probability that I won't wear white pants two days in a row?
13. I roll 5 die for Yahtzee. What is the probability that I get a 5 in a row (12345 or 23456)?
14. What is the probability that a 5-card poker hand contains two pairs (but not 4 of a kind)?